radial coordinate; y_0, dimensionless radius of the elastic core; $\Delta p/l$, pressure drop along the length of the tube; η, dynamic viscosity coefficient; θ, limiting shear stress; τ, tangential stress; $\vartheta = (T - T_w)/(T_0 - T_w)$, dimensionless temperature; $\Phi_n(y)$, eigenfunctions; Pe, Peclet number; $J_0(y)$, $J_{1/3}(y)$, Bessel functions.

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HEAT TRANSFER IN GENERALIZED COUETTE FLOW OF A NONLINEAR VISCOPLASTIC FLUID

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The steady-state heat-transfer problem is solved for dissipative pressure flow of a nonlinear viscoplastic fluid between two parallel isothermal plates, one of which is moving at a constant velocity while the other is stationary.

Let us consider steady-state stabilized flow of a nonlinear viscoplastic fluid between two parallel infinite plates. The upper plate is moving in its own plane at a constant velocity U in the direction of the axis O_x. A constant pressure gradient grad p = A is present in the gap. The gradient can be of mechanical or other origin, such as a magnetic field moving along the channel axis and acting on a ferromagnetic suspension. The orientation of the velocity vector **U** can coincide with the direction of **A** or be opposite to it. This model of generalized Couette flow is valid, for example, for the description of fluid flow in the screw channels of an extruder. We consider the properties of the medium to be independent of the temperature. Constant temperatures are maintained on the plates: $T^{(1)}$ on the lower and $T^{(2)}$ on the upper.

It has been shown [2] that three fully developed flow regimes are possible, depending on the rheological properties of the fluid, the magnitude and direction of the pressure gradient, and the velocity of the upper plate: 1) flow with a quasisolid zone (core) inside the main flow; 2) flow with the core adjacent to one of the plates; 3) flow without any core in the gap.

Accordingly, the equations of motion and thermal energy transport must be solved separately for the different zones and then matched at the interfaces (Fig. 1). Allowance must be made for the fact that dissipation of mechanical into heat energy takes place only in zones I and II, while in zone III the thermal conduction law for solids is realized.

To describe the rheological behavior of the fluid we use the generalized model [1]

$$\tau^{\frac{1}{n}} = \tau_0^{\frac{1}{n}} + (\mu_p \dot{\gamma})^{\frac{1}{m}} \tag{1}$$

with rheological parameters m, n, and μ_p (all real numbers).

Under the given initial assumptions, the problem is stated in the form

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial \tau}{dy}, \tag{2}$$

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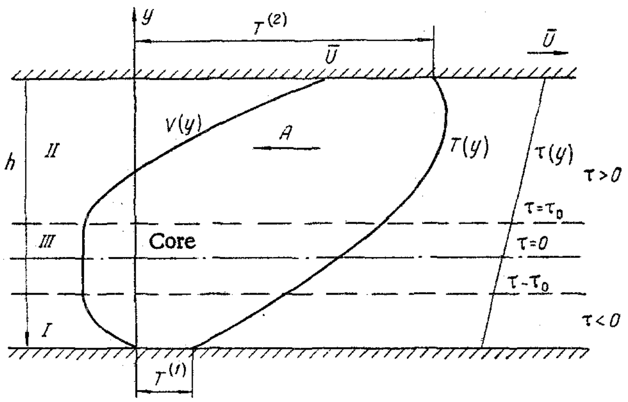

Fig. 1. Illustration of the problem.

$$0 = \lambda \frac{\partial^2 T}{\partial x^2} + \tau \frac{\partial V}{\partial y}. \quad (2)$$

The velocity and temperature boundary conditions are

$$V_1(0) = 0, \; V_2(h) = U, \quad (3)$$

$$T_1(0) = T^{(1)}, \quad T_2(h) = T^{(2)} \quad (4)$$

(the index 1 refers to zone I, the index 2 to zone II, and the index 3 to the quasisolid flow zone III).

Since the properties of the fluid are constant, the equations of motion and thermal energy transport can be solved autonomously. The hydrodynamic problem has already been solved [2]. The expressions for the dimensionless velocity profiles turn out to have the form

$$W_1(\xi) = \frac{1}{\alpha} \sum_{k=0}^{\infty} (-1)^k \Phi_{kmn} [(\xi_0 - \xi)^{\varepsilon_k} - \xi_0^{\varepsilon_k}], \; 0 \leqslant \xi \leqslant \xi_1,$$

$$W_2(\xi) = 1 + \frac{1}{\alpha} \sum_{k=0}^{\infty} (-1)^k \Phi_{kmn} [(\xi - \xi_0)^{\varepsilon_k} - (1 - \xi_0)^{\varepsilon_k}], \quad (5)$$

$$\xi_2 \leqslant \xi \leqslant 1,$$

where

$$\Phi_{kmn} = C_m^k \frac{1}{\varepsilon_k} \beta_0^{\frac{k}{n}}, \; \varepsilon_k = \frac{m + n - k}{n}, \; C_m^k = \frac{m!}{k!\,(m-k)!}.$$

The velocity $W_3(\xi) = W_1(\xi_1) = W_2(\xi_2) = \text{const}$. Expressions are given in the same paper [2] for the characteristic curves separating the zones of different flow regimes in the $0\alpha\beta_0$ plane, along with expressions for determining the values of ξ_0 and the characteristic values of α for each flow regime.

The heat-balance equations are written as follows with consideration for the signs in zones I and II [2]:

$$\text{I}: \lambda \frac{d^2 T_1}{dy^2} - \left[\tau_0^{\frac{1}{n}} + \left(-\mu_p \frac{dV_1}{dy}\right)^{\frac{1}{m}}\right]^n \frac{dV_1}{dy} = 0, \; 0 \leqslant y \leqslant y_1,$$

$$\text{II}: \lambda \frac{d^2 T_2}{dy^2} + \left[\tau_0^{\frac{1}{n}} + \left(\mu_p \frac{dV_2}{dy}\right)^{\frac{1}{m}}\right]^n \frac{dV_2}{dy} = 0, \; y_2 \leqslant y \leqslant h,$$

$$\text{III}: \lambda \frac{d^2 T_3}{dy^2} = 0, \quad y_1 \leqslant y \leqslant y_2.$$

After a few simple manipulations we arrive at the dimensionless representations

$$\frac{d^2\Theta_1}{d\xi^2}+\frac{\varkappa}{\alpha}(\xi_0-\xi)[(\xi_0-\xi)^{\frac{1}{n}}-\beta_0^{\frac{1}{n}}]^m=0,\ 0\leqslant\xi\leqslant\xi_1,$$

$$\frac{d^2\Theta_2}{d\xi^2}+\frac{\varkappa}{\alpha}(\xi-\xi_0)[(\xi-\xi_0)^{\frac{1}{n}}-\beta_0^{\frac{1}{n}}]^m=0,\ \xi_2\leqslant\xi\leqslant 1, \tag{6}$$

$$\frac{d^2\Theta_3}{d\xi^2}=0,\ \xi_1\leqslant\xi\leqslant\xi_2.$$

The boundary conditions and conditions for matching of the solutions at the zone interfaces are

$$\Theta_1(0)=0,\quad \Theta_2(1)=1,$$

$$\Theta_1(\xi_1)=\Theta_3(\xi_1),\quad \Theta_2(\xi_2)=\Theta_3(\xi_2), \tag{7}$$

$$\left.\frac{d\Theta_1}{d\xi}\right|_{\xi_1}=\left.\frac{d\Theta_3}{d\xi}\right|_{\xi_1},\quad \left.\frac{d\Theta_2}{d\xi}\right|_{\xi_2}=\left.\frac{d\Theta_3}{d\xi}\right|_{\xi_2}$$

(the last two conditions are obtained by matching the heat fluxes at the zone interfaces, subject to the additional assumption that the change in structure in transition from the shear zone to the nonshear zone does not alter the value of λ).

The general form of the solution is

$$\Theta_1(\xi)=-\frac{\varkappa}{\alpha}\sum_{k=0}^{\infty}(-1)^k F_{kmn}(\xi_0-\xi)^{\varphi_k}+\frac{\varkappa}{\alpha}(C_1\xi+C_2),$$

$$\Theta_2(\xi)=-\frac{\varkappa}{\alpha}\sum_{k=0}^{\infty}(-1)^k F_{kmn}(\xi-\xi_0)^{\varphi_k}+\frac{\varkappa}{\alpha}(D_1\xi+D_2), \tag{8}$$

$$\Theta_3(\xi)=\frac{\varkappa}{\alpha}(E_1\xi+E_2),$$

where C_i, D_i, and E_i ($i = 1, 2$) are constants of integration and

$$F_{kmn}=C_m^k\frac{n^2}{(m+2n-k)(m+3n-k)}\beta_0^{\frac{k}{n}},$$

$$\varphi_k=\frac{m+3n-k}{n}. \tag{9}$$

We determine the constants of integration from conditions (1.2) and the relations [2]

$$\xi_0-\xi_1=\xi_2-\xi_0=\beta_0,$$

$$\xi_2-\xi_1=2\beta_0. \tag{10}$$

After a series of transformations we obtain for the regime with the core in the flow interior

$$\Theta(\xi)=\begin{cases}\xi+\dfrac{\varkappa}{\alpha}\sum_{k=0}^{\infty}(-1)^kF_{kmn}\Big\{-(\xi_0-\xi)^{\varphi_k}+\xi\Big[\dfrac{2(\beta_0-1)}{\varphi_k}\beta_0^{\varphi_k-1}+\\ \qquad+(1-\xi_0)^{\varphi_k}-\xi_0^{\varphi_k}\Big]+\xi_0^{\varphi_k}\Big\},\ 0\leqslant\xi\leqslant\xi_1,\\ \xi+\dfrac{\varkappa}{\alpha}\sum_{k=0}^{\infty}(-1)^kF_{kmn}\Big\{\xi\Big[\dfrac{2\beta_0-1}{\varphi_k}\beta_0^{\varphi_k-1}+(1-\xi_0)^{\varphi_k}-\xi_0^{\varphi_k}\Big]+\\ \qquad+\xi_0^{\varphi_k}-\dfrac{\xi_0-\beta_0}{\varphi_k}\beta_0^{\varphi_k-1}+\beta_0^{\varphi_k}\Big\},\ \xi_1\leqslant\xi\leqslant\xi_2,\\ \xi+\dfrac{\varkappa}{\alpha}\sum_{k=0}^{\infty}(-1)^kF_{kmn}\Big\{-(\xi-\xi_0)^{\varphi_k}+\xi\Big[\dfrac{2}{\varphi_k}\beta_0^{\varphi_k}+\\ \qquad+(1-\xi_0)^{\varphi_k}-\xi_0^{\varphi_k}\Big]+\Big[\xi_0^{\varphi_k}-\dfrac{2\beta_0^{\varphi_k}}{\varphi_k}-\beta_0^{\varphi_k}\Big\},\ \xi_2\leqslant\xi\leqslant 1.\end{cases}\qquad(11)$$

When the core is situated at the upper wall,

$$\Theta_{uw}(\xi)=\begin{cases}\xi+\dfrac{\varkappa}{\alpha}\sum_{k=0}^{\infty}(-1)^kF_{kmn}\Big[-(\xi_0-\xi)^{\varphi_k}+\xi\Big(\beta_0^{\varphi_k}-\\ \qquad-\dfrac{1-\xi_0+\beta_0}{\varphi_k}\beta_0^{\varphi_k-1}-\xi_0^{\varphi_k}\Big)+\xi_0^{\varphi_k}\Big],\ 0\leqslant\xi\leqslant\xi_1,\\ \xi+\dfrac{\varkappa}{\alpha}\sum_{k=0}^{\infty}(-1)^kF_{kmn}\Big[\xi\Big(\beta_0^{\varphi_k}-\xi_0^{\varphi_k}+\\ \qquad+\dfrac{\xi_0-\beta_0}{\varphi_k}\beta_0^{\varphi_k-1}\Big)+\Big(\xi_0^{\varphi_k}-\beta_0^{\varphi_k}-\\ \qquad-\dfrac{\xi_0-\beta_0}{\varphi_k}\beta_0^{\varphi_k-1}\Big)\Big],\ \xi_1\leqslant\xi\leqslant 1.\end{cases}\qquad(12)$$

When the core is situated at the lower wall,

$$\Theta_{1w}(\xi)=\begin{cases}\xi+\dfrac{\varkappa}{\alpha}\sum_{k=0}^{\infty}(-1)^kF_{kmn}\Big[(1-\xi_0)^{\varphi_k}-\beta_0^{\varphi_k}+\\ \qquad+\dfrac{\xi_0+\beta_0+1}{\varphi_k}\beta_0^{\varphi_k-1}\Big]\xi,\ 0\leqslant\xi\leqslant\xi_2,\\ \xi+\dfrac{\varkappa}{\alpha}\sum_{k=0}^{\infty}(-1)^kF_{kmn}\Big[-(\xi-\xi_0)^{\varphi_k}+\xi\Big((1-\xi_0)^{\varphi_k}-\\ \qquad-\beta_0^{\varphi_k}+\dfrac{\xi_0+\beta_0}{\varphi_k}\beta_0^{\varphi_k-1}\Big)+\beta_0^{\varphi_k}-\dfrac{\xi_0+\beta_0}{\varphi_k}\beta_0^{\varphi_k-1}\Big],\\ \qquad\xi_2\leqslant\xi\leqslant 1.\end{cases}\qquad(13)$$

When the core has ostensibly "transcended" the upper plate (flow without a core), the temperature profile is given by the expression

$$\Theta_u(\xi)=\xi+\frac{\varkappa}{\alpha}\sum_{k=0}^{\infty}(-1)^kF_{kmn}[-(\xi_0-\xi)^{\varphi_k}+\xi((\xi_0-1)^{\varphi_k}-\xi_0^{\varphi_k})+\xi_0^{\varphi_k}],\ 0\leqslant\xi\leqslant 1,\qquad(14)$$

and when the core has "transcended" the lower plate,

$$\Theta_l(\xi) = \xi + \frac{\varkappa}{\alpha} \sum_{k=0}^{\infty} (-1)^k F_{kmn} [-(\xi - \xi_0)^{\varphi_k} + \xi ((1 - \xi_0)^{\varphi_k} -$$

$$- (-\xi_0)^{\varphi_k}) + (-\xi_0)^{\varphi_k}], \quad 0 \leqslant \xi \leqslant 1. \tag{15}$$

The flow regimes (and, accordingly, the expression for calculation of the temperature profile) are determined from the values of (α, β_0) according to [2].

If we neglect the dissipation of mechanical energy in the viscous flow zones I and II, we obtain

$$\Theta(\xi) = \xi, \text{ i.e., } T(y) = T^{(1)} + y(T^{(2)} - T^{(1)}).$$

Then the expressions for the temperature profile can be written in the form

$$\Theta_i(\xi) = \xi + \frac{\varkappa}{\alpha} D_i(\xi), \; i = 1, 2, 3, \tag{16}$$

where $D_i(\xi)$ denotes functions characterizing the temperature increments due to mechanical energy dissipation [expressions for $D_i(\xi)$ are given in Table 1 for selected values of m and n], i.e., the temperature distribution is made up of two parts: 1) a linear part corresponding to the temperature distribution in the moving fluid when dissipation is negligible; 2) a part depending on the dissipative heat-release values.

If $T^{(2)} > T^{(1)}$, then heat input from the fluid to the upper wall is possible, depending on the magnitudes of the velocity U and grad p = A. The change of direction of heat transfer at the upper wall is determined from the condition

$$\left.\frac{d\Theta}{d\xi}\right|_{\xi=1} = 0. \tag{17}$$

If $d\Theta/d\xi|_{\xi=1} < 0$, then heat transfer takes place from the fluid to the upper wall; if $d\Theta/d\xi|_{\xi=1} > 0$, it goes from the upper wall to the fluid (and the upper wall is heated or cooled, respectively).

Condition (12) is equivalent to the condition

$$\mathrm{Br} \cdot \mathrm{Sen} = \alpha\beta_0 \frac{1}{\Sigma_{mn}^{i}(\alpha, \beta_0)}, \tag{18}$$

where $\mathrm{Br} = \mu_p U^2/\lambda(T^{(2)} - T^{(1)})$, $\mathrm{Sen} = \tau_0 h/\mu_p U$ are the Brinkman and Saint-Venant numbers, and $\Sigma_{mn}^{i}(\alpha, \beta_0)$ is a function of α, β_0 depending on the flow regime. For example, in the flow regime with an interior core

$$\Sigma_{mn}^{1}(\alpha, \beta_0) = \sum_{k=0}^{\infty} (-1)^k F_{kmn} \frac{1}{\varphi_k} \left[(1 - \xi_0)^{\varphi_k - 1} - \frac{2}{\varphi_k} \beta_0^{\varphi_k} - (1 - \xi_0)^{\varphi_k} + \xi_0^{\varphi_k} \right].$$

The function Σ_{mn}^{i} depends implicitly on α insofar as ξ_0 is a function of α [2].

For generalized Couette flow of a Newtonian fluid, expression (18) gives

$$\mathrm{Br} = \frac{24\alpha^2}{12\alpha^2 + 4\alpha + 1}. \tag{19}$$

As grad p = A → 0 (i.e., as $\alpha \to \infty$), in which case the generalized Couette flow degenerates into simple Couette flow, Eq. (19) goes over to the well-known result [3]

$$\mathrm{Br} = 2. \tag{20}$$

Graphs of the velocity profiles are given in [2]. We illustrate the temperature profiles in the example of a Shvedov–Bingham medium (Fig. 2).

We define the heat-transfer rate S on the plate as

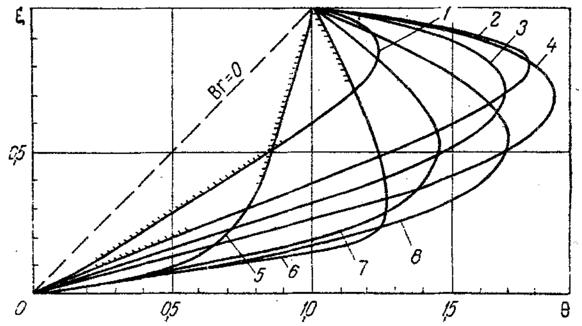

Fig. 2. Temperature curves ($|\varkappa| = 10$, $\beta_0 = 0.2$, $m = n = 1$): 1) $\alpha = 0.1$; 2) 0.3; 3) 0.5; 4) 0.7; 5) −0.1; 6) −0.3; 7) −0.5; 8) −0.7.

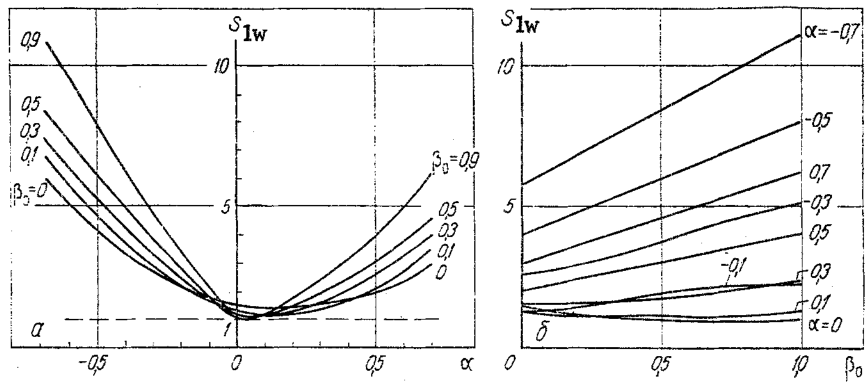

Fig. 3. Heat-transfer rates. a) S_{1w} versus α; b) S_{1w} versus β_0.

$$S = \frac{h}{T^{(2)} - T^{(1)}} \left(\frac{dT}{dy}\right)_{\text{plate}} = \frac{d\Theta}{d\xi}\Big|_{\text{plate}}.$$

The function S has the general form

$$S = 1 + \frac{1}{\alpha\beta_0} \mathrm{Br} \cdot \mathrm{Sen} \cdot P_{mn}(\alpha, \beta_0), \tag{21}$$

where $P_{mn}(\alpha, \beta_0)$ is a function of α and β_0, its form depending on the flow regime. For example, in the case of the upper plate in the flow regime with an interior core

$$P_{mn}(\alpha, \beta_0) = \sum_{k=0}^{\infty} (-1)^k F_{kmn} \left[(1-\xi_0)^{\varphi_k} - \xi_0^{\varphi_k} - \frac{1}{\varphi_k}(1-\xi_0)^{\varphi_k} + \frac{2}{\varphi_k} \beta_0^{\varphi_k - 1} \right].$$

We illustrate the foregoing results in the example of a Shvedov–Bingham medium (Fig. 3). For a fixed dissipation parameter

$$B = \frac{\varkappa}{\alpha} = \frac{1}{\alpha\beta_0} \mathrm{Br} \cdot \mathrm{Sen} = \frac{(Ah^2)^2}{\lambda\mu_p (T^{(2)} - T^{(1)})}$$

and for fixed values of β_0 (corresponding to a fixed value of A for each specific fluid, i.e., for given values of τ_0 and μ_p) the graphs of S_{lw} and S_{uw} in coordinates α, S have a symmetry point (1, 0), i.e.,

$$\beta_0 = \text{fix}, \ S_{1w}(\alpha) + S_{uw}(\alpha) = 2. \tag{22}$$

For fixed values of α (i.e., for fixed U) the graphs have a symmetry axis, namely the line S = 1, i.e.,

TABLE 1. Temperature Profiles for Flow with an Interior Core

m/n	$D_1(\xi)$	$D_2(\xi)$	$D_3(\xi)$
1/1	$\frac{1}{6}\left\{-\frac{1}{2}(\xi_0-\xi)^4+\beta_0(\xi_0-\xi)^3+\xi\left[2\beta_0^3\times(1-\beta_0)+\frac{1}{2}(1-\xi_0)^4-\beta_0(1-\xi_0)^4-\beta_0(1-\xi_0)^3-\frac{1}{2}\xi_0^4+\beta_0\xi_0^3\right]+\frac{1}{2}\xi_0^4-\beta_0\xi_0^3\right\}$	$\frac{1}{6}\left\{-\frac{1}{2}(\xi-\xi_0)^4+\beta_0(\xi-\xi_0)^3+\xi\left[-2\beta_0^4+\frac{1}{2}(1-\xi_0)^4-\beta_0(1-\xi_0)^3-\frac{1}{2}\xi_0^4+\beta_0\xi_0^3\right]+\frac{1}{2}\xi_0^4-\beta_0\xi_0^3+2\beta_0^4\right\}$	$\frac{1}{6}\left\{\xi\left[\beta_0^3(1-2\beta_0)+\frac{1}{2}(1-\xi_0)^4-\beta_0(1-\xi_0)^3-\frac{1}{2}\xi_0^4+\beta_0\xi_0^3\right]+\frac{1}{2}\xi_0^4-\beta_0\xi_0^3+(\xi_0-\beta_0)\beta_0^3+\frac{1}{2}\beta_0^4\right\}$
1/2	$-\frac{4}{35}(\xi_0-\xi)^{\frac{7}{2}}+\frac{1}{6}\beta_0^{\frac{1}{2}}(\xi_0-\xi)^3+\xi\left[\frac{4}{35}\left((1-\xi_0)^{\frac{7}{2}}-\xi_0^{\frac{7}{2}}\right)-\frac{1}{6}\beta_0^{\frac{1}{2}}\left((1-\xi_0)^3-\xi_0^3\right)+\frac{1}{5}\beta_0^{\frac{5}{2}}(1-\beta_0)\right]+\frac{4}{35}\xi_0^{\frac{7}{2}}-\frac{1}{6}\beta_0^{\frac{1}{2}}\xi_0^3$	$-\frac{4}{35}(\xi-\xi_0)^{\frac{7}{2}}+\frac{1}{6}\beta_0^{\frac{1}{2}}(\xi-\xi_0)^3+\xi\left[\frac{4}{35}\left((1-\xi_0)^{\frac{7}{2}}-\xi_0^{\frac{7}{2}}\right)-\frac{1}{6}\beta_0^{\frac{1}{2}}\left((1-\xi_0)^3-\xi_0^3\right)-\frac{1}{5}\beta_0^{\frac{7}{2}}\right]+\frac{4}{35}\xi_0^{\frac{7}{2}}-\frac{1}{6}\beta_0^{\frac{1}{2}}\xi_0^3-\frac{1}{3}\beta_0^{\frac{7}{2}}$	$\xi\left[\frac{4}{35}\left((1-\xi_0)^{\frac{7}{2}}-\xi_0^{\frac{7}{2}}\right)-\frac{1}{6}\beta_0^{\frac{1}{2}}\left((1-\xi_0)^3-\xi_0^3\right)+\frac{1}{10}(1-2\beta_0)\beta_0^{\frac{5}{2}}\right]+\frac{4}{35}\xi_0^{\frac{7}{2}}-\frac{1}{6}\beta_0^{\frac{1}{2}}\xi_0^3+\frac{1}{10}(\beta_0-\xi_0)\beta_0^{\frac{7}{2}}-\frac{11}{210}\beta_0^{\frac{7}{2}}$
2/1	$-\frac{1}{20}(\xi_0-\xi)^5+\frac{1}{6}\beta_0(\xi_0-\xi)^4-\frac{1}{6}\beta_0^2(\xi_0-\xi)^3+\xi\left[\frac{1}{20}\left((1-\xi_0)^5-\xi_0^5\right)-\frac{1}{6}\beta_0\left((1-\xi_0)^4-\xi_0^4\right)+\frac{1}{6}\beta_0^2\left((1-\xi_0)^3-\xi_0^3\right)+\frac{1}{6}(\beta_0-1)\beta_0^4\right]+\frac{1}{20}\xi_0^5-\frac{1}{6}\beta_0\xi_0^4+\frac{1}{6}\beta_0^2\xi_0^3$	$-\frac{1}{20}(\xi-\xi_0)^5+\frac{1}{6}\beta_0(\xi-\xi_0)^4-\frac{1}{6}\beta_0^2(\xi-\xi_0)^3+\xi\left[\frac{1}{20}\left((1-\xi_0)^5-\xi_0^5\right)-\frac{1}{6}\beta_0\times\left((1-\xi_0)^4-\xi_0^4\right)+\frac{1}{6}\beta_0^2\left((1-\xi_0)^3-\xi_0^3\right)+\frac{1}{6}\beta_0^5\right]+\frac{1}{20}\xi_0^5-\frac{1}{6}\beta_0\xi_0^4+\frac{1}{6}\beta_0^2\xi_0^3-\frac{1}{6}\beta_0^5$	$\xi\left[\frac{1}{12}\beta_0^4(2\beta_0-1)+\frac{1}{20}\left((1-\xi_0)^5-\xi_0^5\right)-\frac{1}{6}\beta_0\left((1-\xi_0)^4-\xi_0^4\right)+\frac{1}{6}\beta_0^2\times\left((1-\xi_0)^3-\xi_0^3\right)\right]+\frac{1}{20}\xi_0^5-\frac{1}{6}\beta_0\xi_0^4+\frac{1}{6}\beta_0^2\xi_0^3-(\xi_0-\beta_0)\frac{1}{12}\beta_0^4+\frac{1}{20}\beta_0^5$

TABLE 1. (CONTINUED)

m/n	$D_1(\xi)$	$D_2(\xi)$	$D_3(\xi)$
2/2	$-\frac{1}{12}(\xi_0-\xi)^4+\frac{8}{35}\beta_0^{\frac{1}{2}}(\xi_0-\xi)^{\frac{7}{2}}-$ $-\frac{1}{6}\beta_0(\xi_0-\xi)^3+\xi\Big[\frac{1}{12}((1-\xi_0)^4-\xi_0^4)-$ $-\frac{8}{35}\beta_0^{\frac{1}{2}}((1-\xi_0)^{\frac{7}{2}}-\xi_0^{\frac{7}{2}})+\frac{1}{6}\beta_0\times$ $\times((1-\xi_0)^3-\xi_0^3)+\frac{1}{15}(1-\beta_0)\beta_0^3\Big]+$ $+\frac{1}{12}\xi_0^4-\frac{8}{35}\beta_0^{\frac{1}{2}}\xi_0^{\frac{7}{2}}+\frac{1}{6}\beta_0\xi_0^3$	$-\frac{1}{12}(\xi-\xi_0)^4+\frac{8}{35}\beta_0^{\frac{1}{2}}(\xi-\xi_0)^{\frac{7}{2}}-$ $-\frac{1}{6}\beta_0(\xi-\xi_0)^3+\xi\Big[\frac{1}{12}((1-\xi_0)^4-\xi_0^4)-$ $-\frac{8}{35}\beta_0^{\frac{1}{2}}((1-\xi_0)^{\frac{7}{2}}-\xi_0^{\frac{7}{2}})+\frac{1}{6}\beta_0((1-$ $-\xi_0)^3-\xi_0^3)-\frac{1}{15}\beta_0^4\Big]+\frac{1}{12}\xi_0^4-$ $-\frac{8}{35}\beta_0^{\frac{1}{2}}\xi_0^{\frac{7}{2}}+\frac{1}{6}\beta_0\xi_0^3+\frac{1}{15}\beta_0^4$	$\xi\Big[\frac{1}{12}((1-\xi_0)^4-\xi_0^4)-\frac{8}{35}\beta_0^{\frac{1}{2}}((1-$ $-\xi_0)^{\frac{7}{2}}-\xi_0^{\frac{7}{2}})+\frac{1}{6}\beta_0((1-\xi_0)^3-\xi_0^3)-$ $-\frac{1}{30}\beta_0^3(2\beta_0-1)\Big]+\frac{1}{12}\xi_0^4-$ $-\frac{8}{35}\beta_0^{\frac{1}{2}}\xi_0^{\frac{7}{2}}+\frac{1}{6}\beta_0\xi_0^3+\frac{1}{30}(\xi_0-$ $-\beta_0)\beta_0^3-\frac{3}{140}\beta_0^4$
1/1/2	$-\frac{4}{35}(\xi_0-\xi)^{\frac{7}{2}}+\frac{1}{6}\beta_0^{\frac{1}{2}}(\xi_0-\xi)^3+$ $+\xi\Big[\frac{4}{35}((1-\xi_0)^{\frac{7}{2}}-\xi_0^{\frac{7}{2}})-\frac{1}{6}\beta_0^{\frac{1}{2}}((1-$ $-\xi_0)^3-\xi_0^3)-\frac{\beta_0-1}{5}\beta_0^{\frac{5}{2}}\Big]+\frac{4}{35}\xi_0^{\frac{7}{2}}-$ $-\frac{1}{6}\beta_0^{\frac{1}{2}}\xi_0^3$	$-\frac{4}{35}(\xi-\xi_0)^{\frac{7}{2}}+\frac{1}{6}(\xi-\xi_0)^3+\xi\Big[\frac{4}{35}\times$ $\times((1-\xi_0)^{\frac{7}{2}}-\xi_0^{\frac{7}{2}})-\frac{1}{6}\beta_0^{\frac{1}{2}}((1-\xi_0)^3-$ $-\xi_0^3)-\frac{1}{5}\beta_0^{\frac{7}{2}}\Big]+\frac{4}{35}\xi_0^{\frac{7}{2}}-\frac{1}{6}\beta_0^{\frac{1}{2}}\xi_0^3+$ $+\frac{1}{5}\beta_0^{\frac{7}{2}}$	$\xi\Big[(1-2\beta_0)\cdot\frac{1}{10}\beta_0^{\frac{5}{2}}+\frac{4}{35}((1-\xi_0)^{\frac{7}{2}}-$ $-\xi_0^{\frac{7}{2}})-\frac{1}{6}\beta_0^{\frac{1}{2}}((1-\xi_0)^3-\xi_0^3)\Big]+$ $+\frac{4}{35}\xi_0^{\frac{7}{2}}-\frac{1}{6}\beta_0^{\frac{1}{2}}\xi_0^3+(\beta_0-\xi_0)\times$ $\times\frac{1}{10}\beta_0^{\frac{5}{2}}+\frac{11}{210}\beta_0^{\frac{7}{2}}$
1/2/1	$-\frac{12}{105}((\xi_0-\xi)-\beta_0)^{\frac{7}{2}}-\frac{2}{25}\beta_0[(\xi_0-\xi)-$ $-\beta_0]^{\frac{5}{2}}+\xi\Big[\frac{12}{105}((1-\xi_0-\beta_0)^{\frac{7}{2}}-(\xi_0-\beta_0)^{\frac{7}{2}})+$ $+\frac{2}{25}\beta_0((1-\xi_0-\beta_0)^{\frac{5}{2}}-(\xi_0-\beta_0)^{\frac{5}{2}})\Big]+$ $+\frac{12}{105}(\xi_0-\beta_0)^{\frac{7}{2}}+\frac{2}{25}(\xi_0-\beta_0)^{\frac{5}{2}}$	$-\frac{12}{105}((\xi-\xi_0)-\beta_0)^{\frac{7}{2}}-\frac{2}{25}((\xi-\xi_0)-$ $-\beta_0)^{\frac{5}{2}}+\xi\Big[\frac{12}{105}((1-\xi_0-\beta_0)^{\frac{7}{2}}-(\xi_0-$ $-\beta_0)^{\frac{7}{2}})+\frac{2}{25}\beta_0((1-\xi_0-\beta_0)^{\frac{5}{2}}-(\xi_0-$ $-\beta_0)^{\frac{5}{2}})\Big]+\frac{12}{105}(\xi_0-\beta_0)^{\frac{7}{2}}+\frac{2}{25}(\xi_0-\beta_0)^{\frac{5}{2}}$	$\xi\Big[\frac{12}{105}((1-\xi_0-\beta_0)^{\frac{7}{2}}-(\xi_0-\beta_0)^{\frac{7}{2}})+$ $+\frac{2}{25}\beta_0((1-\xi_0-\beta_0)^{\frac{5}{2}}-(\xi_0-$ $-\beta_0)^{\frac{5}{2}})\Big]+\frac{12}{105}(\xi_0-\beta_0)^{\frac{7}{2}}+$ $+\frac{2}{25}(\xi_0-\beta_0)^{\frac{5}{2}}$
1/2/1/2	$-F(\xi_0-\xi)+\xi[F(1-\xi_0)-F(\xi_0)]+F(\xi_0)$	$-F(\xi-\xi_0)+\xi[F(1-\xi_0)-F(\xi_0)]+F(\xi_0)$ $F(Y)=\frac{Y}{8}(2Y^2-5\beta_0^2)\sqrt{Y^2-\beta_0^2}+$ $+\frac{3}{8}\beta_0^4\ln\lvert Y+\sqrt{Y^2-\beta_0^2}\rvert$	$\xi[F(1-\xi_0)-F(\xi_0)]+F(\xi_0)+$ $+\frac{3}{8}\beta_0^4\ln\lvert\beta_0\rvert$

$$|\alpha| = \text{fix}, \; S_{1w}(\alpha, \beta_0) = -S_{uw}(-\alpha, \beta_0). \tag{23}$$

Moreover, for fixed values of α such that $|\alpha| \geq 1/2$ (i.e., for flow regimes without an interior core) the dependence of S on β_0 exhibits a linear behavior.

The dependence of S on B is linear for any m and n and any flow regimes.

We give special mention to the case $T^{(1)} = T^{(2)}$, i.e., where both plates are maintained at the same temperature. The dimensionless temperature is given by the relation $\vartheta = (T - T^{(1)})/T^{(1)}$, and the solutions for ϑ follow directly from the solutions for Θ, i.e.,

$$\vartheta_i(\xi) = \Theta_i(\xi) - \xi,$$

$$S_{T^{(1)}=T^{(2)}} = S_{T^{(1)}\neq T^{(2)}} - 1.$$

On the basis of these relations all the results obtained for unequal temperatures remain valid in this case.

NOTATION

Dimensioned quantities: λ, thermal conductivity; U, velocity of upper plate; grad p = A, pressure gradient; τ, shear stress; τ_0, ultimate shear stress; μ_p, analog of plastic viscosity; m, n, nonlinearity parameters of flow curve; h, width of channel; y_1, y_2, boundaries of flow core; y, vertical coordinate; V(y), flow velocity; T(y), temperature of medium; $\dot{\gamma} = dV/dy$, shear velocity. Dimensionless quantities: W = V/U, flow velocity; $\xi = y/h$, vertical coordinate; ξ_1, ξ_2, boundaries of core; ξ_0, coordinate of plane of zero shear stress; $\Theta = (T - T^{(1)})/(T^{(2)} - T^{(1)})$, dimensionless temperature; $\alpha = \mu_p U/(Ah)^{m/n} h$, $\beta_0 = \tau_0/Ah$, $\varkappa = AUh^2/\lambda \cdot (T^{(2)} - T^{(1)})$, parameters; $Br = \mu_p U^2/\lambda(T^{(2)} - T^{(1)})$, generalized Brinkman number; $Sen = \tau_0 h/\mu_p U$, generalized Saint-Venant number; S, dimensionless heat-transfer rate on either plate.

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